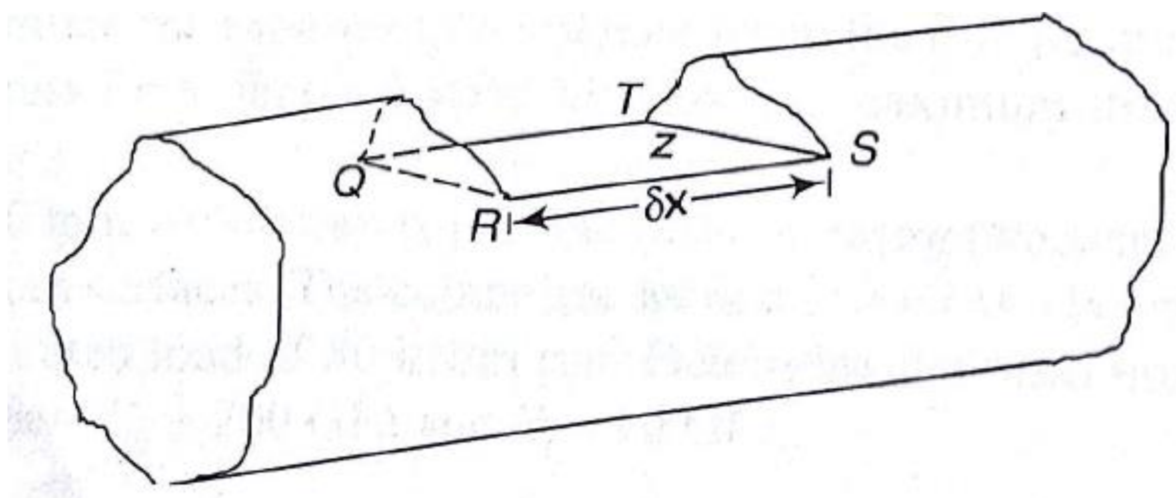
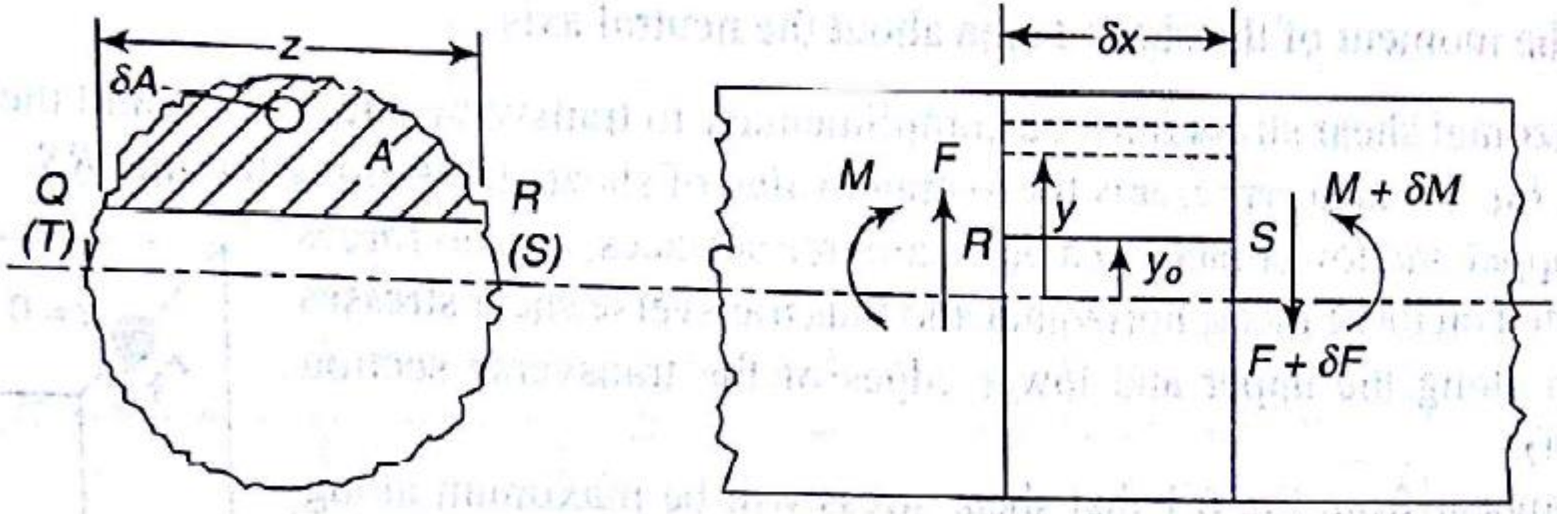
Module 4 b

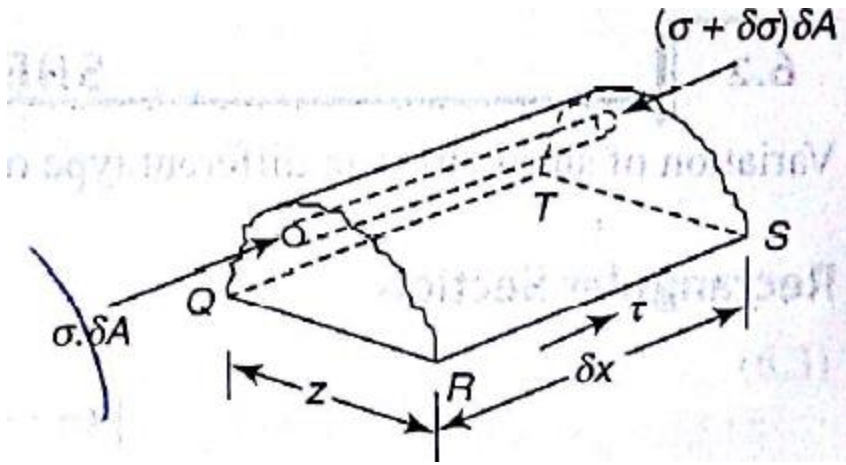
Shear stress in beams

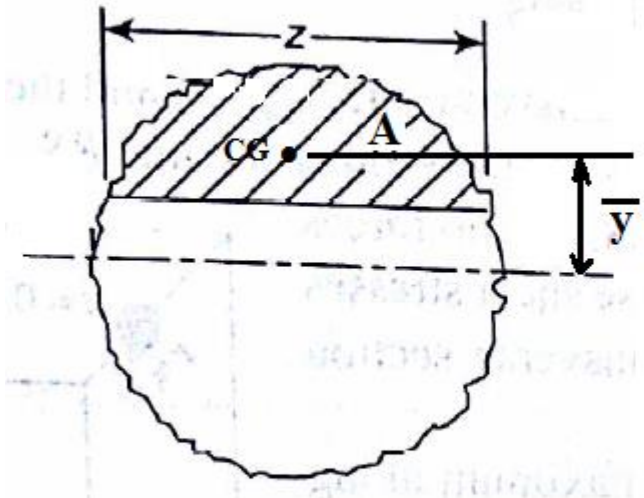


T
Z
S
Q
R
δx



(a) (b)


(σ + δσ)δA
T
S
Q
σ.δA
τ
z
R
δx



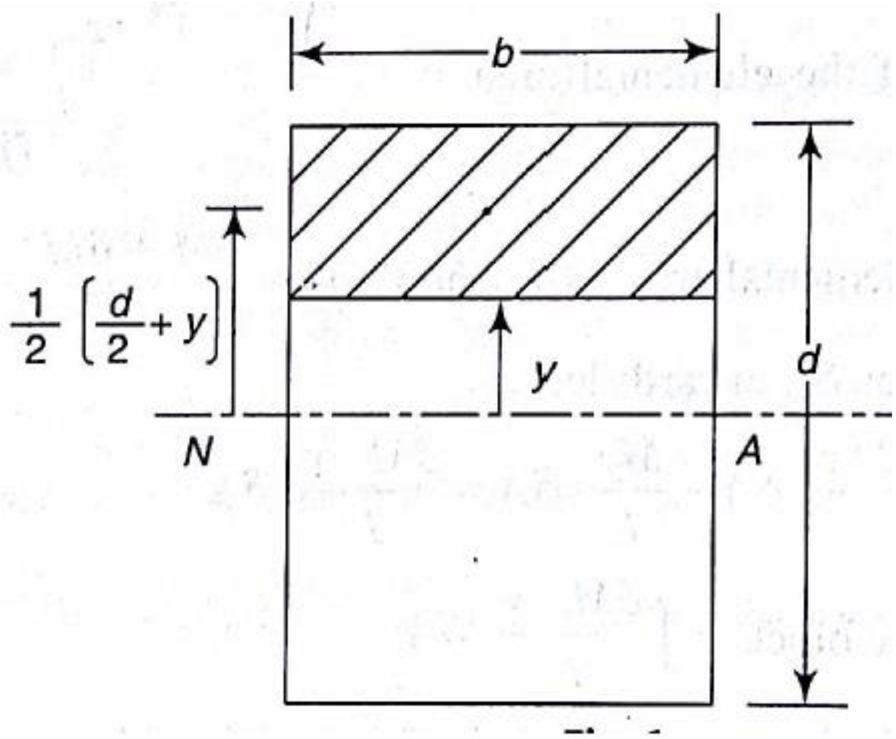
$$\tau = F \cdot \frac{A\overline{y}}{zI}$$

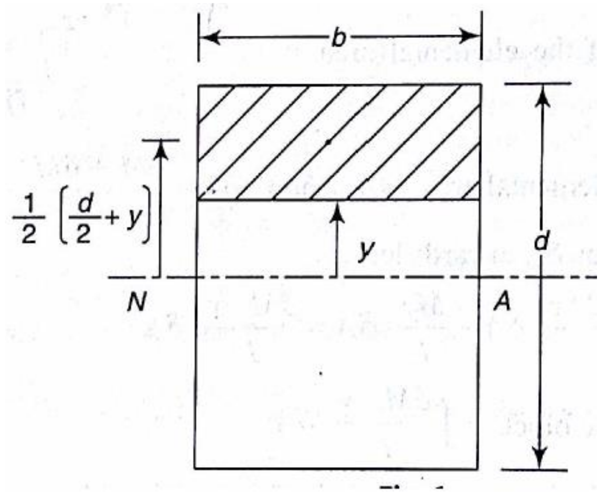
— z is the actual width of the section at the position where τ is to be calculated
— I is the total moment of inertia about the neutral axis

— $A\overline{y}$ is the moment of the shaded area about the neutral axis

Shear stress variation in different sections

- (1) Rectangular section





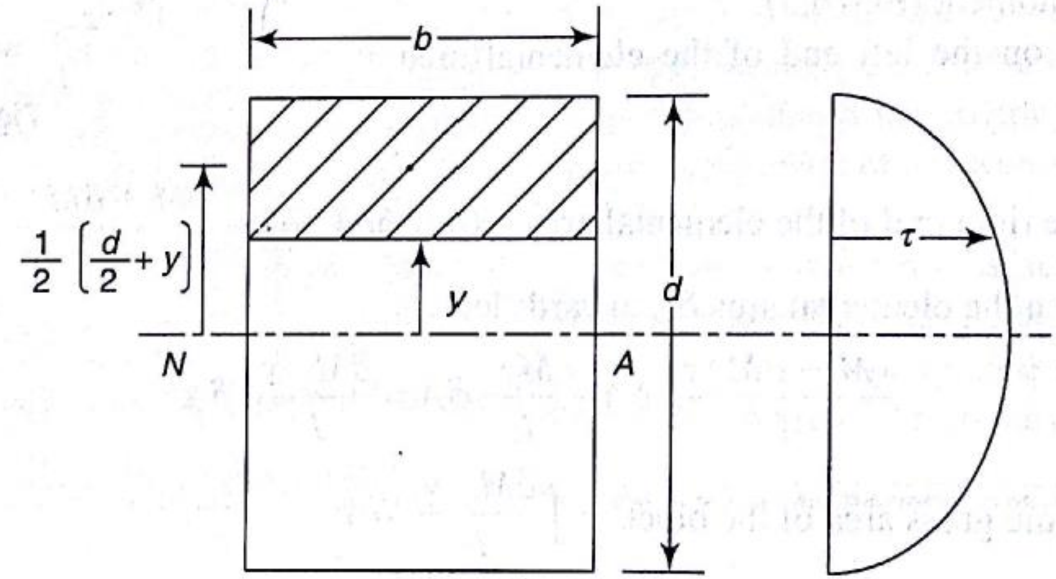
At a distance y from neutral axis (Fig. 6.5),

$$\tau = F \cdot \frac{A\bar{y}}{zI} = F \cdot \frac{b\left(\frac{d}{2} - y\right) \cdot \left(\frac{d/2 + y}{2}\right)}{b \cdot \left(\frac{bd^3}{12}\right)} = \frac{6F}{bd^3} \cdot \left(\frac{d^2}{4} - y^2\right)$$

This indicates that there is parabolic variation of shear stress with y.

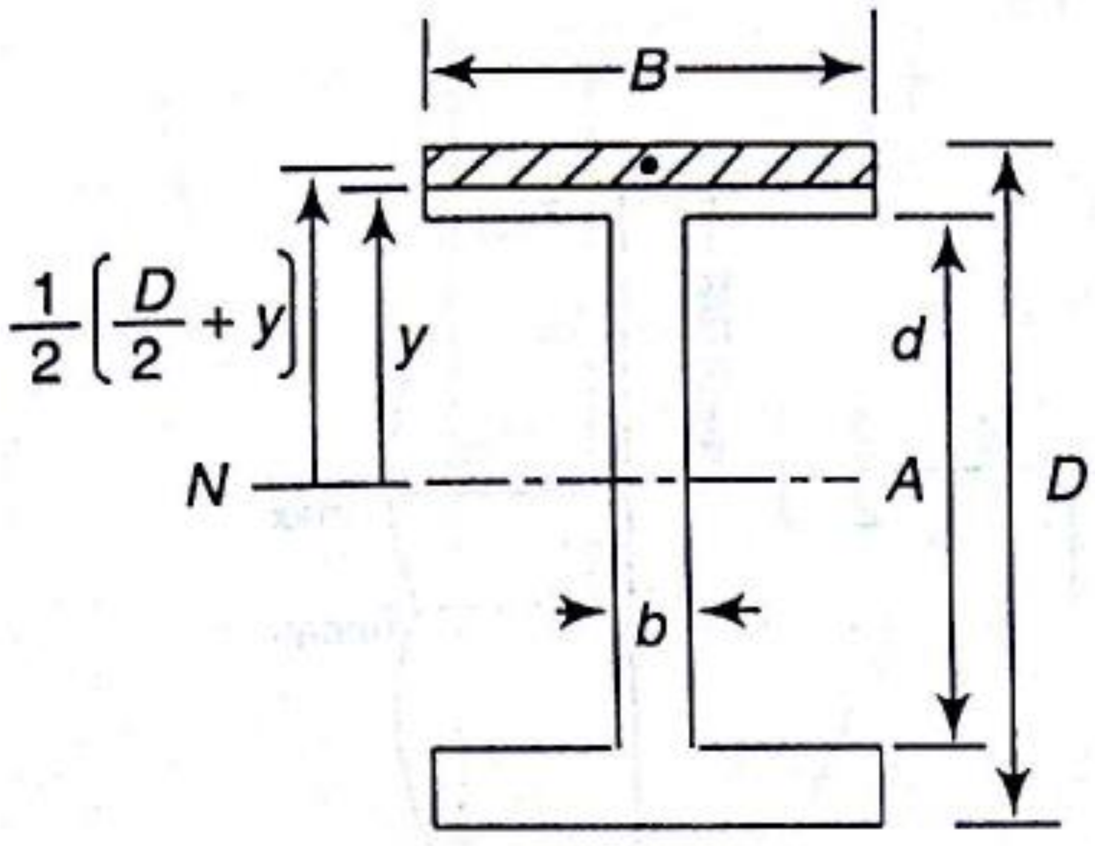
At neutral axis ($y = 0$), Shear stress $= \frac{3}{2} \cdot \frac{F}{bd}$; it is the maximum shear stress.

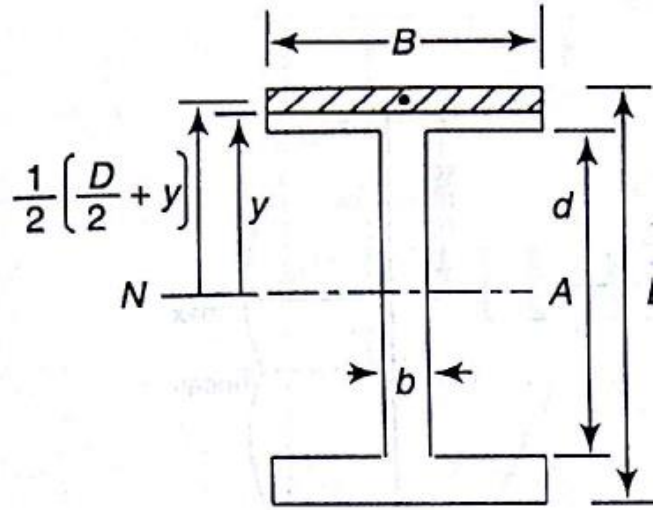
Usually, F/bd is known as the *mean stress* and thus $\tau_{max} = 1.5\, \tau_{mean}$



b
$\frac{1}{2}\left[\frac{d}{2}+y\right]$
y
d
τ
N
A

- (2) I section





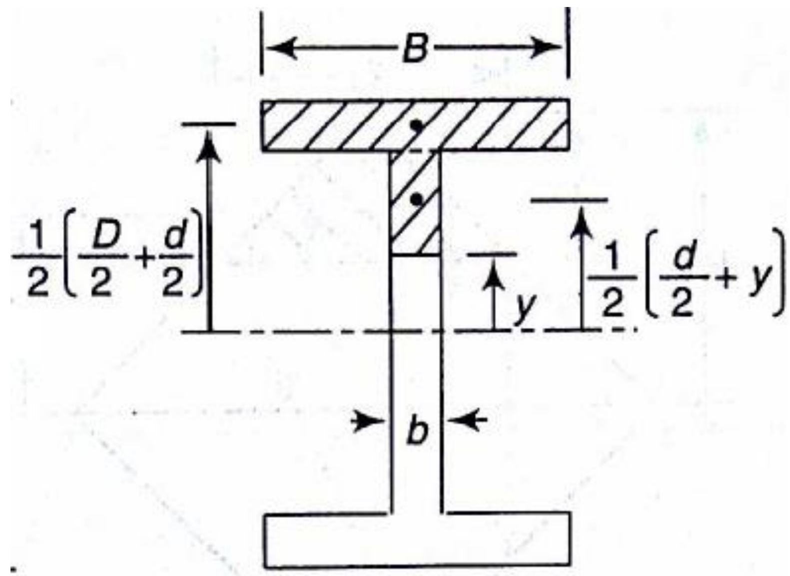
(a)

In the flange, at a distance y from neutral axis (Fig. 6.6a),

$$\tau_f = F \cdot \frac{A\bar{y}}{zI} = F \cdot \frac{B\left(\frac{D}{2}-y\right)\left[\frac{1}{2}\left(\frac{D}{2}+y\right)\right]}{B \cdot I} = \frac{F}{2I} \cdot \left(\frac{D^2}{4} - y^2\right)$$

At $y = D/2$, $\sigma = 0$

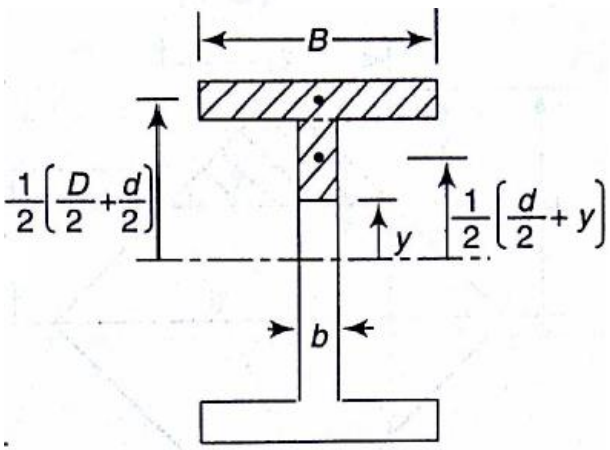
At $y = d/2$, $\tau = \frac{F}{8I}(D^2 - d^2)$



B
1/2(D/2 + d/2)
y
1/2(d/2 + y)
b

In the web, at a distance y from neutral axis (Fig. 6.6b),

$$\tau_w = F \cdot \frac{A\bar{y}}{zI} = F \cdot \frac{A\bar{y}\text{ (for flange)} + A\bar{y}\text{(for web)}}{zI}$$



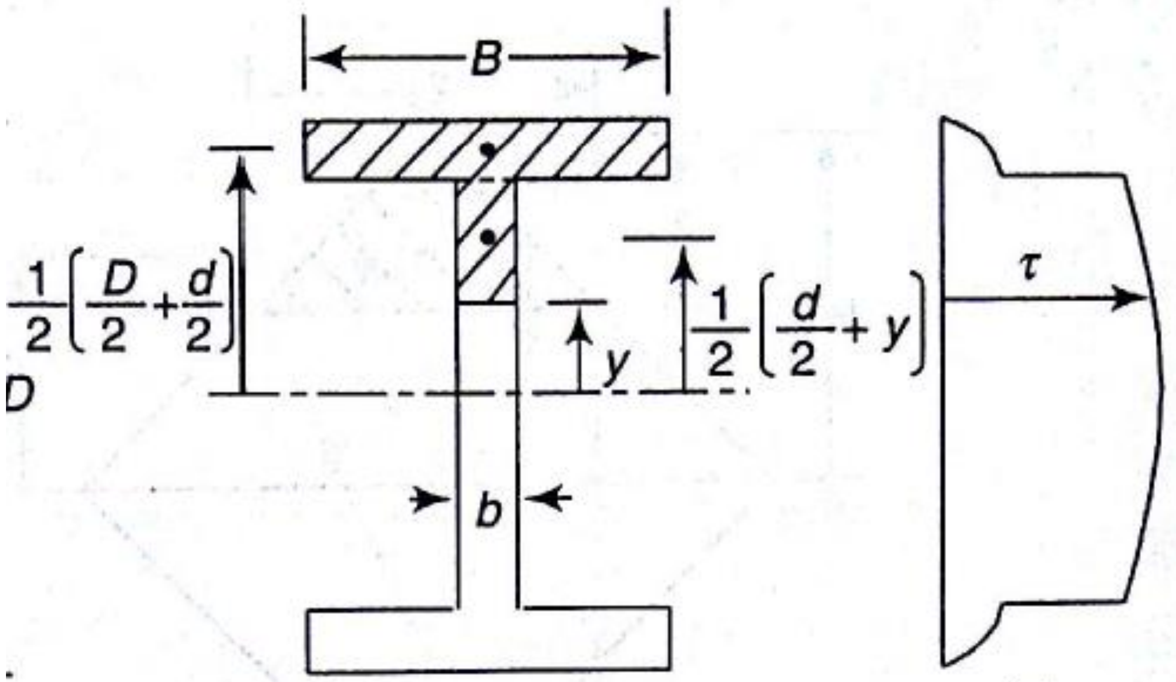
$$= F \cdot \frac{B\left(\frac{D}{2}-\frac{d}{2}\right)\left[\frac{1}{2}\left(\frac{D}{2}+\frac{d}{2}\right)\right] + b\left(\frac{d}{2}-y\right)\left[\frac{1}{2}\left(\frac{d}{2}+y\right)\right]}{b \cdot I}$$

$$= \frac{F}{bI} \cdot \left[B\left(\frac{D-d}{2}\right)\left(\frac{D+d}{4}\right) + \frac{b}{2}\left(\frac{d-2y}{2}\right)\left(\frac{d+2y}{2}\right)\right]$$

$$= \frac{F}{8I}\left[\frac{B}{b}(D^2-d^2) + (d^2-4y^2)\right]$$

Maximum shear stress $= \frac{F}{8I} \cdot \left[\frac{B}{b}(D^2-d^2) + d^2\right]$ at the neutral axis ($y = 0$)

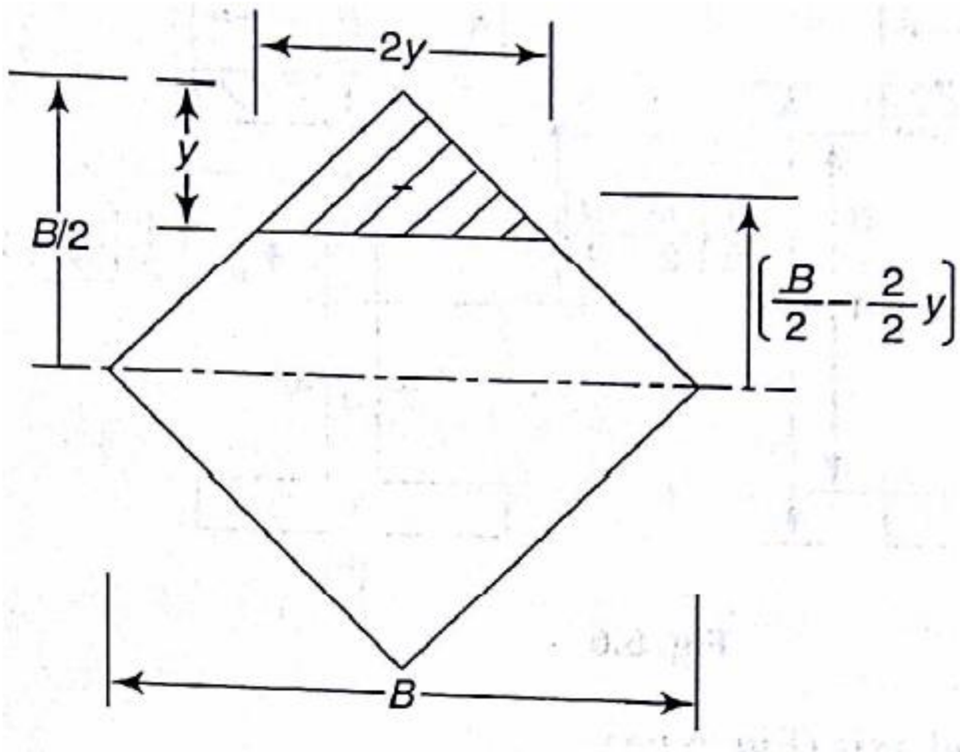
At the top of web, $y = d/2$, $\frac{F}{8I} \cdot \left[\frac{B}{b}(D^2-d^2)\right] = \frac{B}{b} \cdot \tau_f$



B
$\frac{1}{2}\left(\frac{D}{2}+\frac{d}{2}\right)$
D
y
$\frac{1}{2}\left(\frac{d}{2}+y\right)$
b
τ

(b) (c)

- (3) Square with diagonal Horizontal



Moment of inertia about the neutral axis, $I = 2\left[\frac{B \cdot (B/2)^3}{12}\right] = \frac{B^4}{48}$

$$F \cdot \frac{A\bar{y}}{zI} = F \cdot \frac{\frac{2y \cdot y}{2}\left(\frac{B}{2} - \frac{2}{3}y\right)}{(2y) \cdot \left(\frac{B^4}{48}\right)} = \frac{24Fy}{B^4} \cdot \left(\frac{B}{2} - \frac{2}{3}y\right)$$

$$= \frac{4Fy}{B^4}(3B - 4y) \text{ i.e. it is parabolic.}$$

- At $y = 0$, $\tau = 0$
- At neutral axis, $y = B/2$, $\tau = \frac{2F}{B^2}$

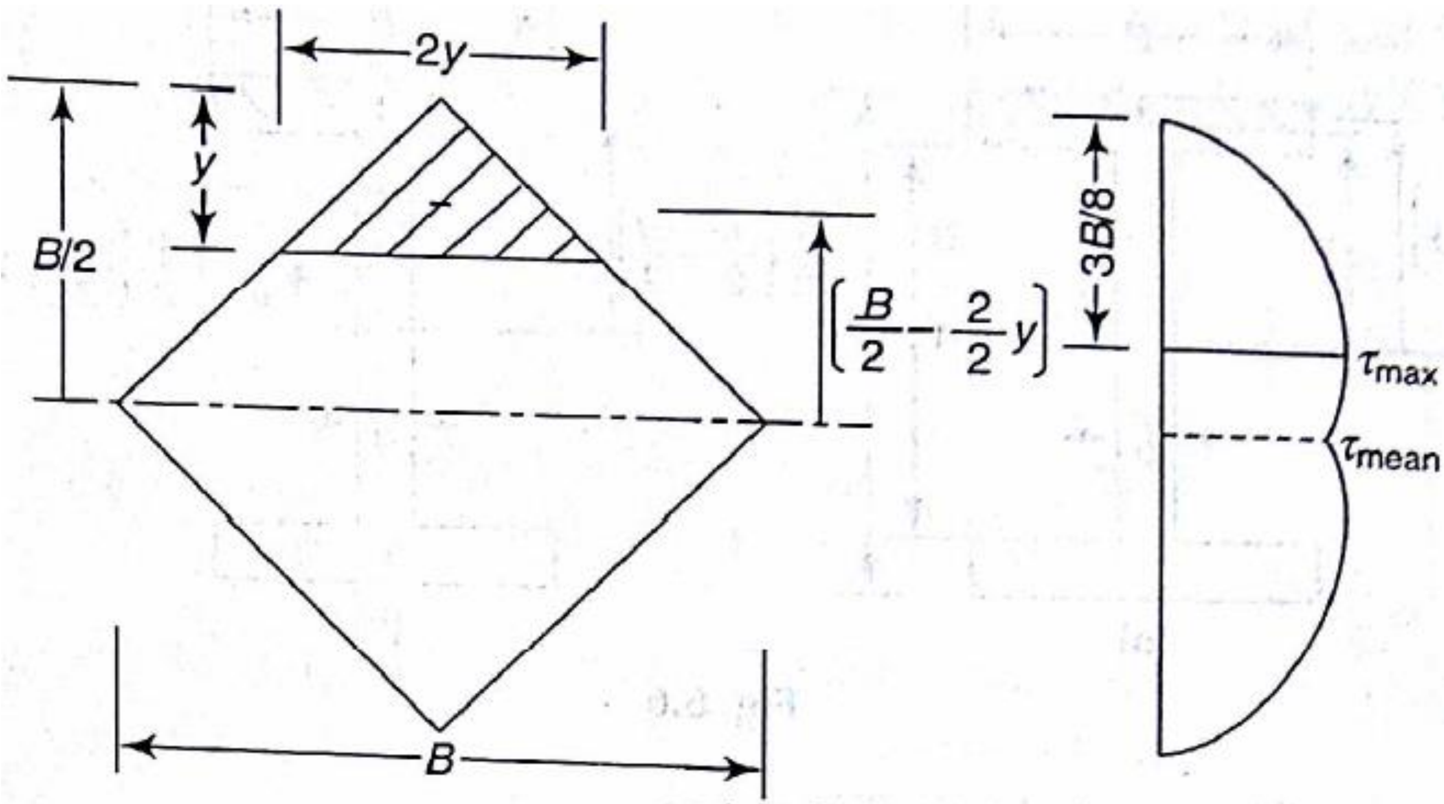
  If b is the side of the square, $B = \sqrt{2}b$,

- At neutral axis, $\tau = \frac{2F}{(\sqrt{2}b)^2} = \frac{F}{b^2} = \frac{F}{\text{area}} = \tau_{\text{mean}}$

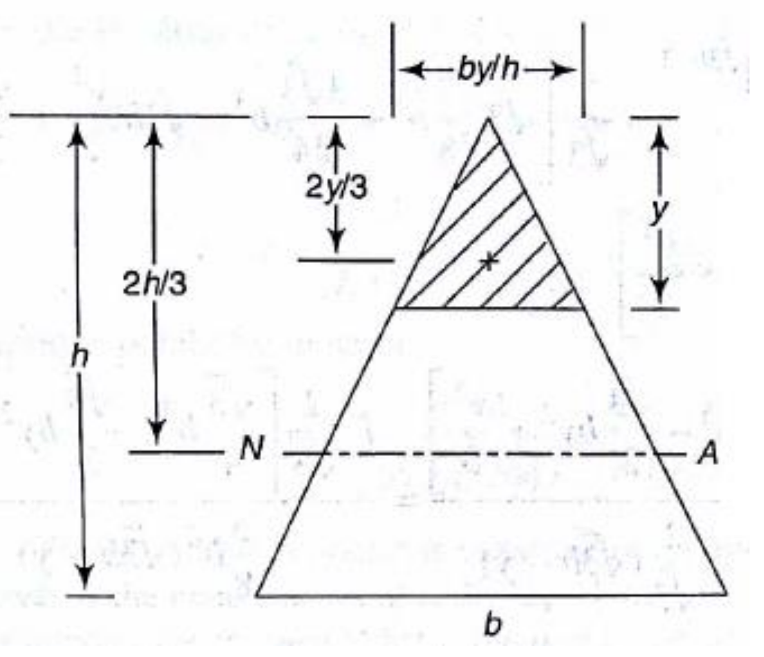
For maximum value, $\frac{d\tau}{dy} = \frac{d}{dy}(3By - 4y^2) = 0$ or $3B - 8y = 0$

or $y = \frac{3}{8}B = \frac{3\sqrt{2}b}{8}$

$$\tau_{max} = \frac{4Fy}{B^4}(3B - 4y) = \frac{4F(3/8)B}{B^4}\left(3B - \frac{4\times 3}{8}B\right) = \frac{9F}{4B^2} = \frac{9F}{4(\sqrt{2}b)^2} = \frac{9F}{8b^2} = \frac{9}{8}\tau_{mean}$$


2y
y
B/2
$\left[\frac{B}{2} - \frac{2}{2}y\right]$
3B/8
τ_{max}
τ_{mean}
B

- (4) Triangular section

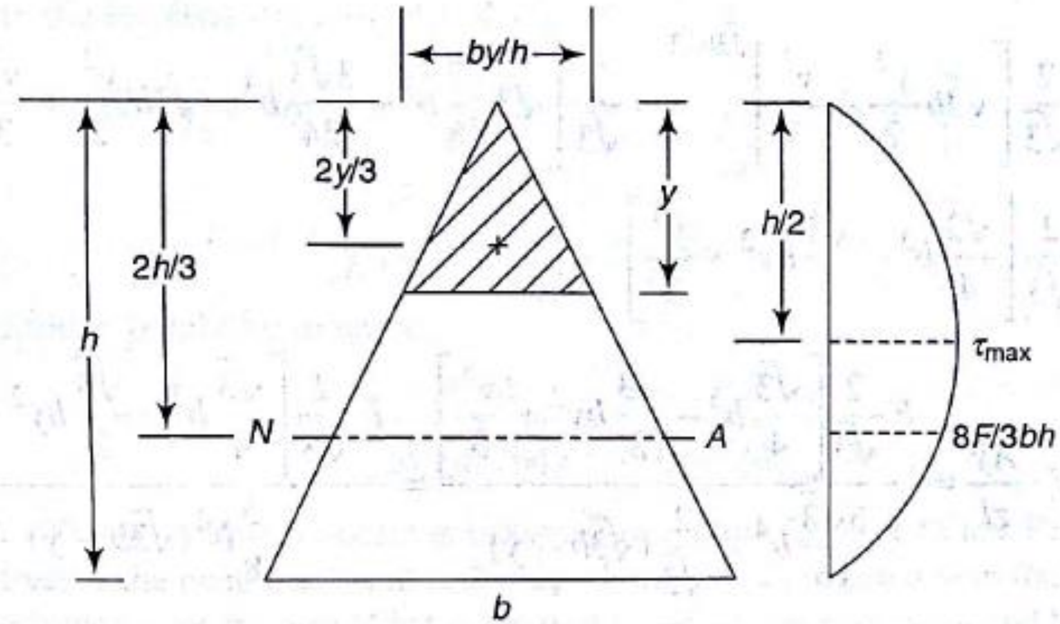


$$\tau = F \cdot \frac{A\bar{y}}{zI} = \frac{F\left(\frac{1}{2} \cdot y \cdot \frac{by}{h}\right)\left(\frac{2}{3}h - \frac{2}{3}y\right)}{\frac{by}{h} \cdot \frac{bh^3}{36}} = \frac{12Fy}{bh^3}(h - y)$$

For maximum value, $\frac{d\tau}{dy} = \frac{d}{dy}(hy - y^2) = 0$ or $h - 2y = 0$ or $y = h/2$

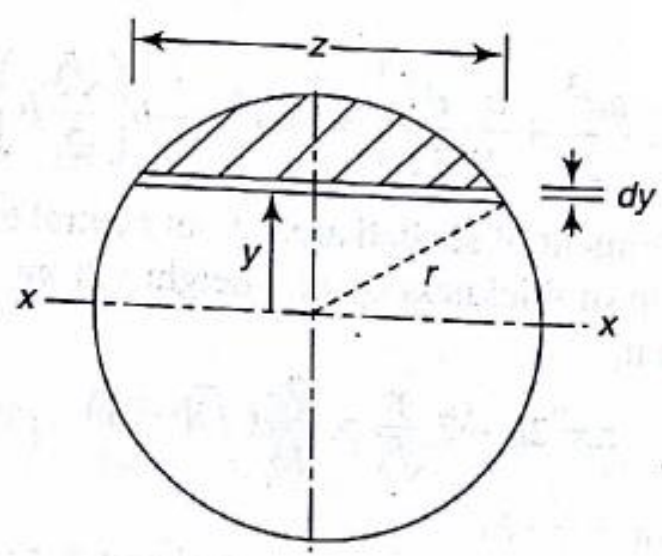
$$\tau_{\text{max}} = \frac{12Fh/2}{bh^3}\left(h - \frac{h}{2}\right) = \frac{3F}{bh} = \frac{3}{2} \cdot \frac{F}{bh/2} = 1.5\tau_{\text{mean}}$$

At neutral axis, $\tau = \frac{12F(2h/3)}{bh^3}\left(h - \frac{2h}{3}\right) = \frac{8F}{3bh}$



by/h
2y/3
2h/3
h
y
h/2
τmax
N
A
8F/3bh
b

- (5) Circular section



$$z = 2\sqrt{r^2 - y^2} \quad \text{or} \quad z^2 = 4(r^2 - y^2)$$

or $$2z \cdot dz = -8y\,dy$$

or $$y\,dy = -z \cdot dz/4 \qquad \text{(i)}$$

Now, $A\bar{y}$ = Moment of shaded area about neutral axis

Consider a strip of thickness δy at a height y from neutral axis and parallel to it,

Area of the strip = $z \cdot \delta y$

Moment of elementary area about neutral axis = $z \cdot \delta y \cdot y$

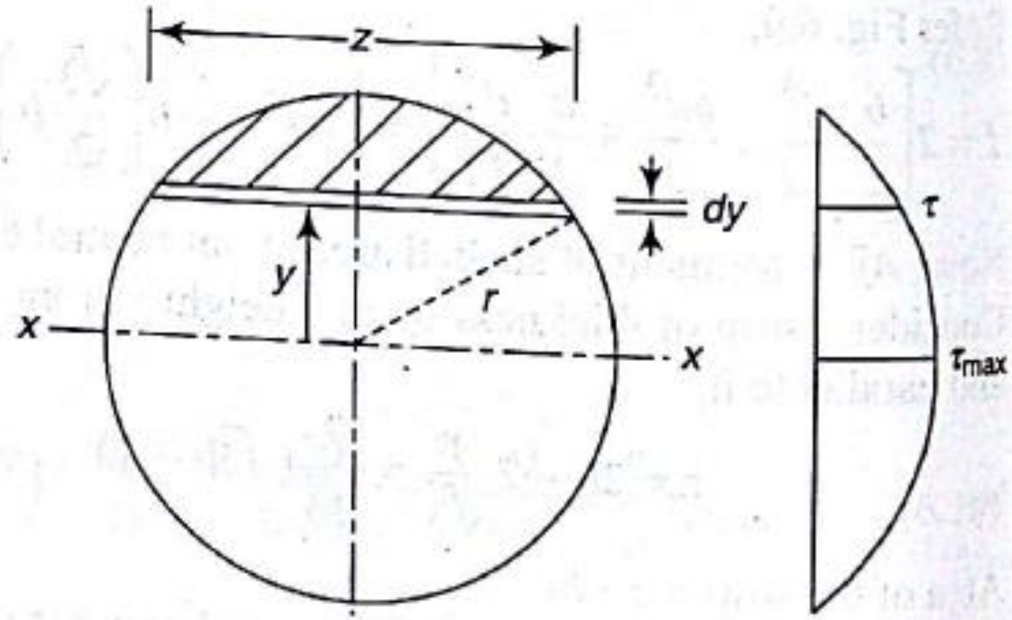
Moment of whole of the shaded area about neutral axis,

$$A\bar{y} = \int_y^r z \cdot dy \cdot y = -\frac{1}{4}\int_z^0 z \cdot z \cdot dz = \frac{1}{4}\int_0^z z^2 \cdot dz = \frac{z^3}{12}$$

$$\tau = F \cdot \frac{1}{zI}\frac{z^3}{12} = F \cdot \frac{1}{I}\frac{z^2}{12} = \frac{F}{3I}(r^2 - y^2)$$

Thus, shear stress variation is parabolic in nature.

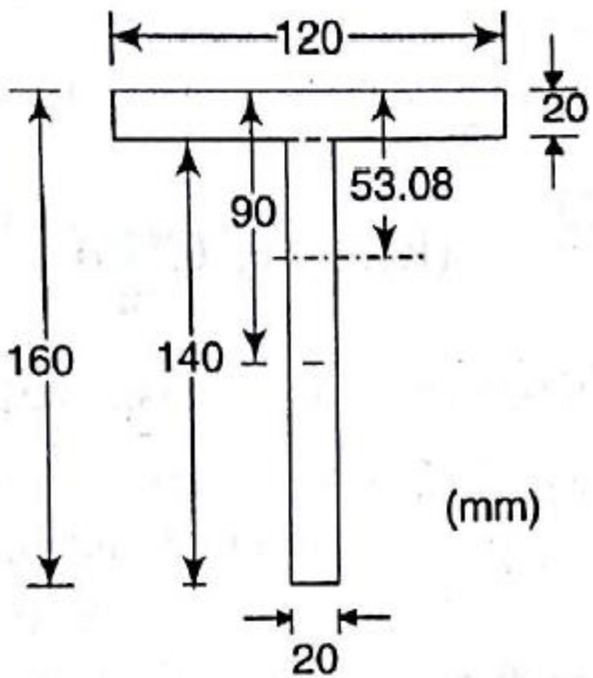
$$\tau_{\max(y=0)} = F \cdot \frac{d^2/4}{3\,(\pi d^4/64)} = \frac{4}{3}\frac{F}{(\pi d^2/4)} = \frac{4}{3}\tau_{av}$$

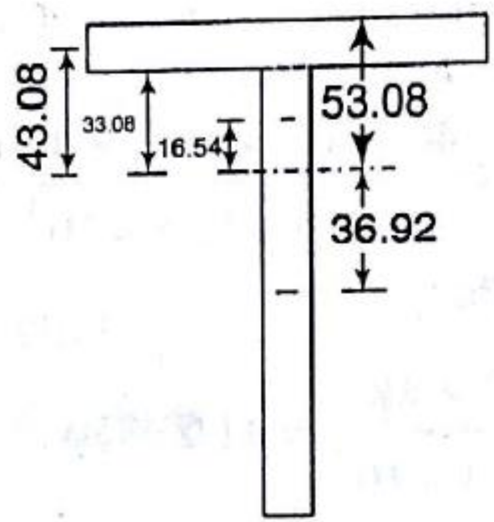

z
dy
y
r
x
x
τ
τ_{max}

Example 6.2 A simply supported beam of 2-m span carries a uniformly distributed load of 140 kN per m over the whole span. The cross-section of the beam is a *T*-section with a flange width of 120 mm, web and flange thickness of 20 mm and overall depth of 160 mm. Determine the maximum shear stress in the beam and draw the shear stress distribution for the section.

$$\text{Reaction at each end of the beam} = \frac{140 \times 2}{2} = 140 \text{ kN}$$

$$\text{Thus maximum shear force} = 140 \text{ kN}$$





Taking moments about the top edge,

$$\bar{y} = \frac{120 \times 20 \times 10 + 140 \times 20 \times 90}{120 \times 20 + 140 \times 20} = 53.08 \text{ mm}$$

$$I = \underbrace{\frac{120(20)^3}{12} + 120 \times 20 \times 43.08^2}_{\text{(flange)}} + \underbrace{\frac{20(140)^3}{12} + 20 \times 140 \times 36.92^2}_{\text{(web)}}$$

$$= 12.924 \times 10^6 \text{ mm}^4$$

Shear stress

Shear stress in the flange at the junction of flange and web

$$= F \cdot \frac{A\bar{y}}{zI} = \frac{140\,000 \times (120 \times 20) \times 43.08}{12.924 \times 10^6 \times 120} = 9.333 \text{ MPa}$$

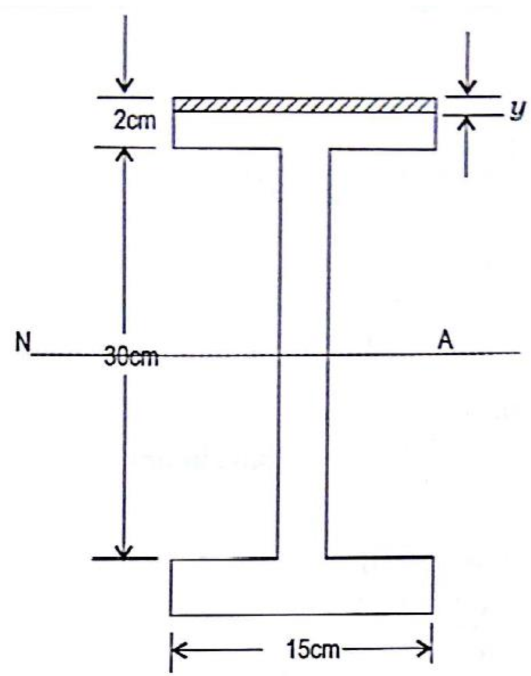
Shear stress in the web at the junction $= 9.333 \times \frac{120}{20} = 56$ MPa

Maximum shear stress (at neutral axis) $= \frac{140\,000 \times [120 \times 20 \times 43.08 + (20 \times 33.08) \times 16.54]}{12.924 \times 10^6 \times 20} = 61.93$ MPa

An I section has the following dimensions. Flanges : 15 cm × 2 cm; Web : 30 cm × 2 cm. Find the maximum shearing stress developed in the beam for a shearing force of 10 kN. Draw the stress variation.

Solution

$$\tau = \frac{FA\bar{y}}{Ib} \quad ; \qquad I = \frac{15 \times 34^3}{12} - \frac{13 \times 30^3}{12} = 19880 \text{ cm}^4$$



In the top flange area, $b = 15$ cm.

$$\therefore \tau \text{ at bottom of top flange} = \frac{10 \times (15 \times 2 \times 16)}{19880 \times 15} = 0.0161 \text{ kN/cm}^2$$

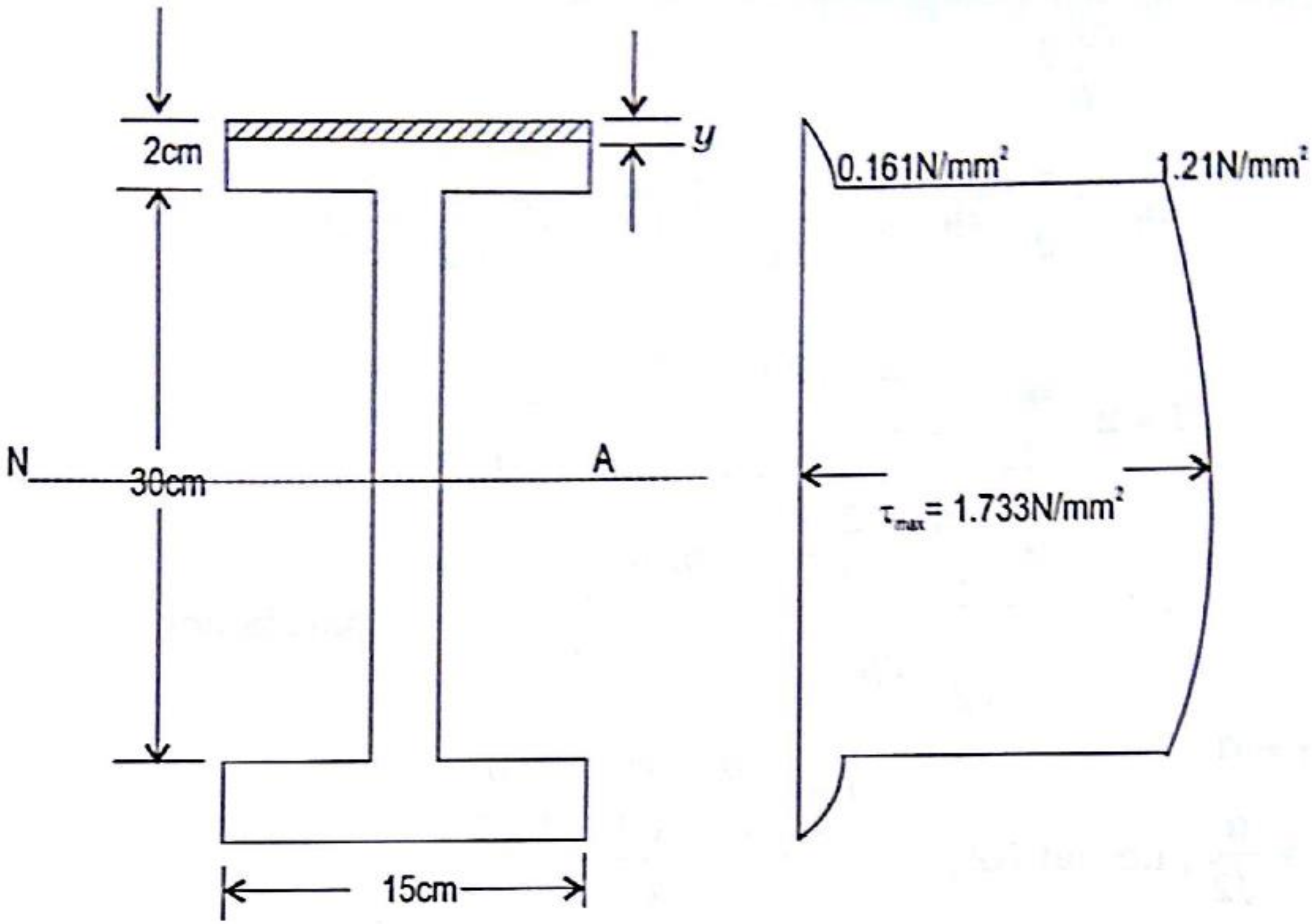
$$= 0.161 \text{ N/mm}^2$$

$$\tau \text{ at top of web } (b = 2 \text{ cm}) = \frac{10 \times 15 \times 2 \times 16}{19880 \times 2} = 0.121 \text{ kN/cm}^2$$

$$= 1.21 \text{ N/mm}^2$$

$$\tau_{max} \text{ at NA} = \frac{10[15 \times 2 \times 16 + 15 \times 2 \times 7.5]}{19880 \times 2} = 0.1773 \text{ kN/cm}^2$$

$$= 1.773 \text{ N/mm}^2$$



2cm
y
0.161N/mm²
1.21N/mm²
N
A
30cm
τmax = 1.733N/mm²
15cm

A T-section beam 300 mm deep and 150 mm wide has flange and web thickness of 30 mm. The length of the beam is 6 m and is simply supported at its ends. It carries a UDL of 5 kN/m over its entire length. In addition to the UDL, it carries a concentrated load of 3kN at its middle. Draw the shear stress distribution diagram for the beam.

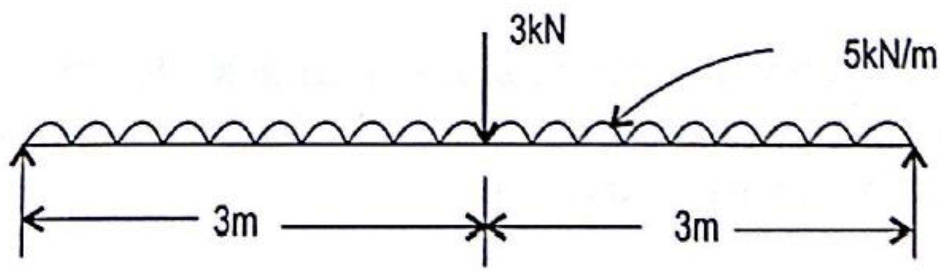


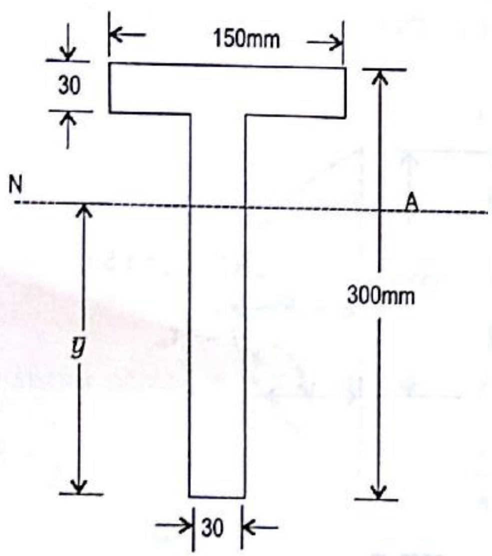
Figure E7.2a

$$\text{Max SF at supports} = \frac{5 \times 6 + 3}{2} = 16.5 \text{ kN} \quad \text{(due to symmetrical loading)}$$

First let us calculate position of CG and NA and MI about NA.

$$\bar{y} = \frac{150 \times 30 \times 285 + 270 \times 30 \times 135}{150 \times 30 + 270 \times 30} = 188.57 \text{ mm}$$

$$MI = \frac{30 \times 270^3}{12} + 30 \times 270(188.57 - 135)^2$$



$$+ \frac{150 \times 30^3}{12} + 150 \times 30(285 - 188.57)^2$$

$$= 114635525.5 \text{ mm}^4$$

$$= 1.146 \times 10^8 \text{ mm}^4$$

$$\tau = \frac{FA\bar{y}}{Ib}$$

Shear stress at bottom of top flange

$$= \frac{16.5 \times 10^3 (150 \times 30 \times (285 - 188.57))}{1.146 \times 10^8 \times 150}$$

$$= 0.4165 \text{ N/mm}^2$$

Shear stress at top of web $= \dfrac{16.5 \times 10^3 \times 150 \times 30(285 - 188.57)}{1.146 \times 10^8 \times 30}$

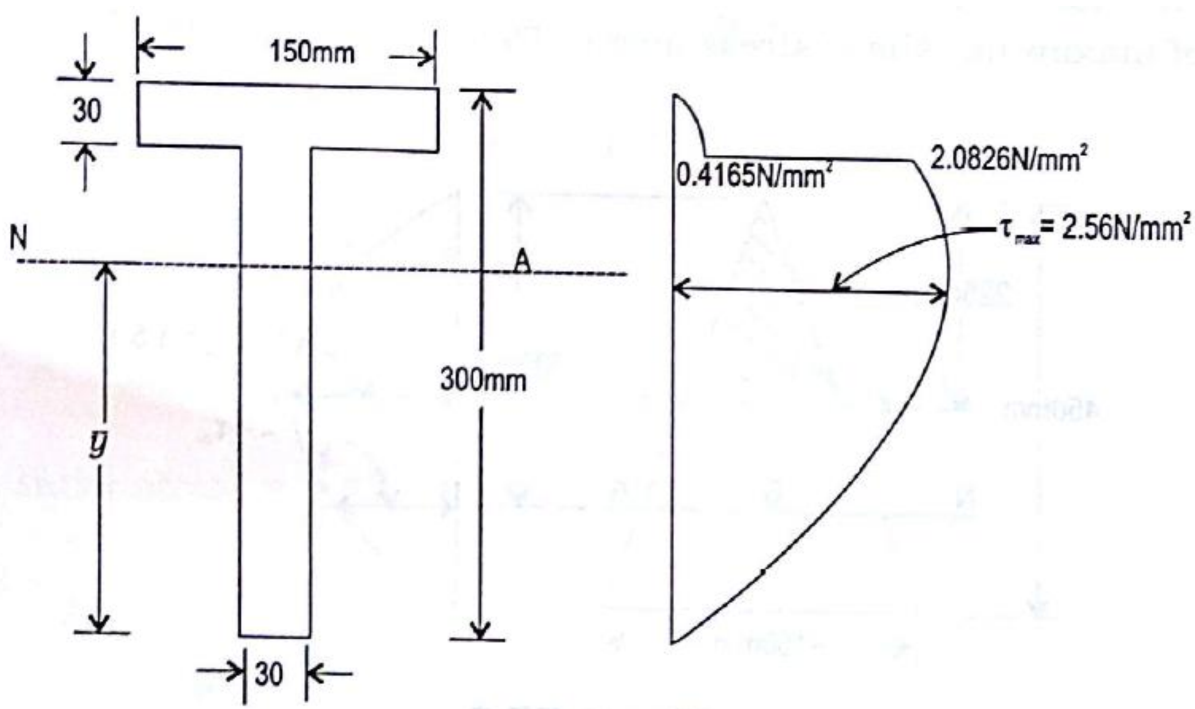
$$= 2.0826 \text{ N/mm}^2$$

Shear stress at NA

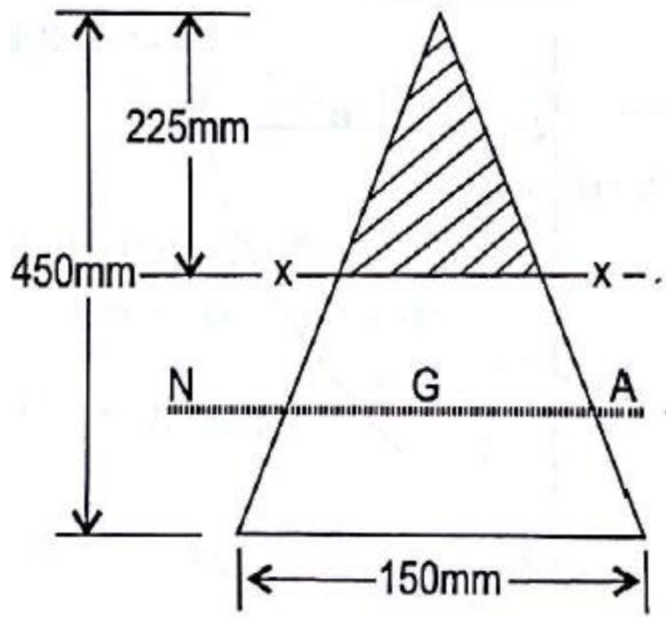
$$= \frac{16.5\times10^3\times\left(150\times30(285-188.57)+\frac{(300-30-188.57)^2}{2}\times30\right)}{1.146\times10^8\times30}$$

$= 2.56\ \text{N/mm}^2$

Shear stress is zero at top and bottom fibre and maximum at NA.



A beam having the cross-section of an isosceles triangle has a horizontal base width b=150 mm, and height h=450 mm. It is subjected to shear force of 60 kN at a section Find the horizontal shear stress at the neutral axis, where does the intensity of maximum, shear stress occur? Determine its value.



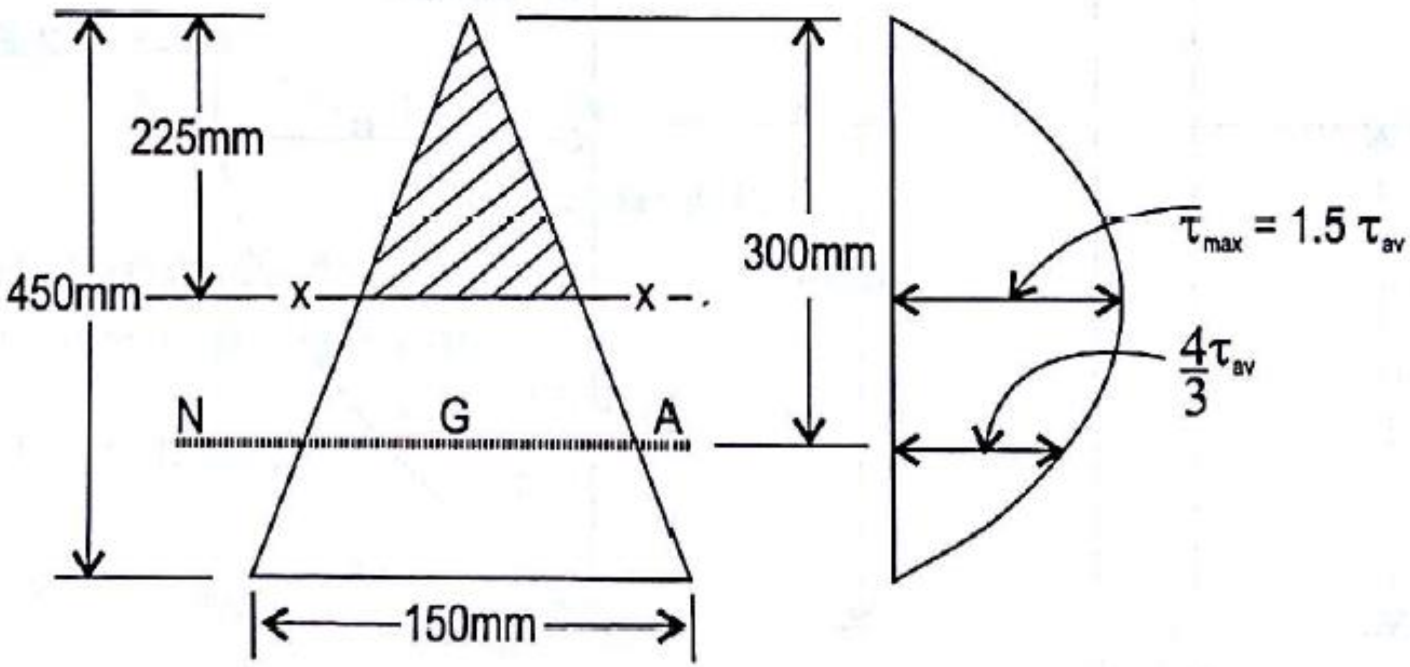
We know that in a triangular section, maximum shear stress occurs at $\frac{h}{2}$.

$$\therefore \tau_{max} = 1.5\tau_{av} = \frac{F}{\frac{1}{2}bh}$$

$$= \frac{1.5 \times 60 \times 10^3}{\frac{1}{2} \times 150 \times 450} = 2.67 \text{N/mm}^2$$

At neutral axis, *ie*, at two third height from the centre, shear stress is $\frac{4}{3}\tau_{av}$.

$$\therefore \tau_{NA} = \frac{4}{3}\left[\frac{60 \times 10^3}{\frac{1}{2} \times 150 \times 450}\right] = 2.37 \text{N/mm}^2$$



225mm
450mm
X
X
300mm
N
G
A
150mm
$\tau_{max} = 1.5\ \tau_{av}$
$\frac{4}{3}\tau_{av}$